

071606T4ABT

AUTOBODY TECHNOLOGY LEVEL 6

ENG/OS/ABT/CC/01/6/A

Apply Engineering Mathematics

July/August 2024



**TVET CURRICULUM DEVELOPMENT, ASSESSMENT AND CERTIFICATION
COUNCIL (TVET CDACC)**

WRITTEN

TIME: 3 HOURS

INSTRUCTIONS TO THE CANDIDATE

1. *This paper consists of **TWO** sections: **A** and **B**.*
2. *Attempt **ALL** the questions in section **A** and any **THREE** questions from section **B**.*
3. *Marks for each question are as indicated.*
4. *You are provided with a separate answer booklet.*
5. *Do not write on the question paper.*

This paper consists of FOUR (4) printed.

**Candidates should check the question paper to ascertain that all pages
are printed as indicated and that no questions are missing**

Turn over

SECTION A: (40 MARKS)**Attempt ALL the questions in this section**

- Solve the equation; $\log_{4x}(\frac{1}{2}) - \log_{(\frac{x}{2})} \frac{1}{2} = \frac{3}{4}$ (5 Marks)
- Given the complex numbers $Z_1 = 2 + 5j$, $Z_2 = 1 - 3j$ and $Z_3 = 2 + 3j$. Determine $4Z_1^2 + 3Z_2Z_3$ in form of $a + bj$. (3 Marks)
- Determine the fourth term in the binomial expansion of $(x + 3y)^{12}$ and find its value when $x = \frac{1}{2}$ and $y = \frac{1}{3}$ (4 Marks)
- Prove the following identity (4 Marks)

$$\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$
- A solid bar of a metal is to be melted and casted into a bowl in the shape of a hemisphere. If the internal radius of the bowl is to be 200mm and its thickness is 3.0mm. Determine the length of the metal bar if its diameter is (4 Marks)
- The Table 1 below shows the marks scored by motive engineering students in a control system exam.

Table 1

Marks	0-3	4-7	8-11	12-15	16-19	20-23	24-27	28-31
No of students	3	5	a	8	7	b	4	3

If the mean mark is 15.5, determine the values of a and b . (5 Marks)

- Given the vectors $A = i + 4j - 2k$ and $B = 2i - j + 3k$. Determine $|A \times B|$. (4 Marks)
- Given the matrices;

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ -2 & 4 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 & 3 \\ -5 & -3 & 0 \\ 1 & 2 & -2 \end{bmatrix}$$

Find $AB - A$. (3 Marks)

- Determine the values of constants P and Q in the equation;

$$P \cosh 3x - Q \sinh 3x = e^{3x} + 5e^{-3x}$$
 (4 Marks)
- Determine the first four terms of the power series for $\cos x$ (4 Marks)

SECTION B: (60 MARKS)**Attempt any THREE questions in this section.**

11.

- a) Given the two vectors $P = 2i + 3j + k$ and $Q = 4i + j - 2k$. Determine;
- Unit vector perpendicular to both vectors. (4 Marks)
 - The angle between the vectors. (4 Marks)
- b) If vector F and scalar Φ are given by $F = 2x^2yz i + 4xyz j - 2xyz^2 k$ and $\Phi = 4x^2y^3z^4$. Determine at point $(1, -1, 2)$; (12 Marks)
- $\nabla\Phi$
 - $\nabla \cdot F$
 - $\nabla \times F$

12.

- a) Solve the differential equation $(x + x^2) \frac{dy}{dx} = y^2$. (8 Marks)
- b) Use the method of undetermined coefficients to solve the differential equation:
 $9 \frac{d^2y}{dx^2} - 12 \frac{dy}{dx} + 4y = 3x - 1$, when $x \frac{dy}{dx} = -\frac{4}{3}$. (12 Marks)

13.

- a) Use the Laplace transforms of the second derivative to prove that:
 $L(\cos at) = \frac{s}{s^2 + a^2}$. (4 Marks)
- b) Determine the inverse Laplace transforms of $\frac{4s - 15}{s^2 - 6s + 13}$. (4 Marks)
- c) A dynamic system is modelled by the differential equation:
 $\frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 4y = e^{-3t}$
 Use the Laplace transforms to determine an expression for $y(t)$, given that $y = 0$, $t = 0$ and $\frac{dy}{dt} = 2$. (12 Marks)

14.

- a) Show that the polar form of the Cartesian equation $x^2 + xy + y^2 - 2x = 0$ is given by
 $r = \frac{2 \cos \theta}{1 + \sin \theta \cos \theta}$. (5 Marks)
- b) Show that $\operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1 - x^2}}{x} \right)$. (5 Marks)

c) Given that $\sin(x - \alpha) = 2\cos(x + \alpha)$,

i). Show that $\tan x = \frac{2 + \tan \alpha}{1 + 2 \tan \alpha}$ (5 Marks)

ii). Hence solve the equation:

$\sin(x - \frac{\pi}{4}) = 2\cos(x + \frac{\pi}{4})$, for values of x between 0° and 360° inclusive. (5 Marks)